Relay feedback test used for process identification and PID tuning controller by genetic algorithms

Identificación de procesos y sintonización de controladores PID mediante una prueba de relé usando un algoritmo genético simple

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ABSTRACT

In this article, a proposal to solve two control problems from multiple point identification process frequency response of linear models, using a relay closed loop is presented. The identified points are used, in one case a PID controller tuning, and the other application deals with transfer function modeling problem. Both problems are stated as a nonlinear least squares unconstrained minimization problem. The optimization problem is solved with a simple genetic algorithm.
1 INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are widely used in many control systems. In process control, more than ninety-five percent of the control loops are of PI or PID type [1, 2]. Since Ziegler and Nichols [3] proposed their empirical method to tune PID controllers, to date, many relevant methods to improve the tuning of PID controllers has been reported at the control literature, one of them is a tutorial given by Hang et al. [4].

The relay feedback auto-tuning method was proposed by Astrom and Hugglund [5], and was one of the first to be commercialized and has remained attractive owing to its simplicity. In this method, the estimation of critical point over nyquist curve is enough to tune a PID controller. In recently studies, it has been shown that the multiple identified points allow better PID tuning controller [6, 7]. This work presents two applications of the multiple-point identification method, reported by Wang et al. [6, 8], in order to tune PID controllers and, on the other hand, to obtain transfer function coefficients. The control problem is posed as a nonlinear least squares unconstrained problem.

A genetic algorithm is proposed to solve the optimization problem. The same methodology can be used for both cases: PID tuning and transfer function modeling. Nonlinear least squares methods involve an iterative improvement of parameter values in order to reduce the sum of the squares of the errors between the function and the measured data points. Problems of this type occur when fitting model functions to experimental data. The Levenberg-Marquardt algorithm [9-11], is the most common method for nonlinear least-squares minimization, nevertheless it can suffer from a slow convergence, and it is possible to find only a local minimum [10].

The PID’s designed with this method takes into account the effect of the sensitivity function values of the closed-loop system as a measure of robustness against possible variations in the parameters of the plant [1]. The proposed plants in this article cover a wide range of cases: stable, with short and long dead times, wit real and complex poles, and with positive and negative zeros, which are representative of the automatic control literature [4, 5].

The contents of the paper are described as follows: In section 2 the basic definitions of a nonlinear least squares unconstrained minimization problem, use of relay transient as well as a simple genetic algorithm procedure are shown. Section 3 presents applications of the multiple point identification method to a PID controller tuning and to transfer function modeling. Conclusions are contained in section 4.

2 BASIC CONCEPTS

2.1 Unconstrained minimization problem

In a large number of practical problems, the objective function $f(\mathbf{x})$ is a sum of squares of nonlinear functions

$$f(\mathbf{x}) = \frac{1}{2} \sum_{j=1}^{m}(r_j(\mathbf{x}))^2 = \frac{1}{2} \|\mathbf{r}(\mathbf{x})\|^2_2$$

that needs to be minimized. We consider the following problem

$$\min_{\mathbf{x}} f(\mathbf{x}) = \min_{\mathbf{x}} \frac{1}{2} \sum_{j=1}^{m}(r_j(\mathbf{x}))^2$$

This is an unconstrained nonlinear least squares minimization problem. It is called least squares because the sum of squares of these functions is the quantity to be minimized. Problems of this type occur when fitting model functions to data: if $\varphi(\mathbf{x}; t)$ represents the model function with $t$ as an independent variable, then each $r_j(\mathbf{x}) = \varphi(\mathbf{x}; t) - y_j$, where $\varphi(t, y)$ is the given set of data points [10, 11].

2.2 Use of relay transient

It was shown by Wang et al. [6] who propose a method that can identify multiple points simultaneously under one relay test. Other important researches on the topic can be found at [12-15]. For a standard relay feedback system in figure 3, the process input $u(t)$ and output $y(t)$ are recorded from the initial time until the system reaches a stationery oscillation. $U(t)$ and $Y(t)$ are not integrable since they do not die down in finite time. They cannot be directly transformed to frequency response meaningfully using FFT. A decay exponential $e^{-\alpha t}$ is then introduced to form

$$\hat{u}(t) = u(t)e^{-\alpha t}$$

and

$$\hat{y}(t) = y(t)e^{-\alpha t}$$
such that $u(t)$ and $y(t)$ will decay to zero exponentially as $t$ approaches infinity. Applying the Fourier transform to (3) and (4) yields

$$\bar{U}(t) = \int_{0}^{\infty} \hat{u}(t)e^{-jwt} \, dt = U(jw+\alpha)$$

and

$$\hat{Y}(t) = \int_{0}^{\infty} \hat{y}(t)e^{-jwt} \, dt = Y(jw+\alpha)$$

For a process $G(s)=Y(s)/U(s)$, at $s=jw+\alpha$, one has

$$G(jw+\alpha) = \frac{Y(jw+\alpha)}{U(jw+\alpha)} = \frac{\hat{Y}(jw)}{\hat{U}(jw)} \quad (5)$$

$\hat{Y}(jw)$ and $\hat{U}(jw)$ can be computed at discrete frequencies with the standard FFT technique [6, 8]. Therefore, the shifted process frequency response $G(jw+\alpha)$ can be obtained from (5). To find $G(jw)$ from $G(jw+\alpha)$, we first take the inverse FFT of $G(jw+\alpha)$ as

$$\tilde{g}(kT) = \text{FFT}^{-1}(G(jw+\alpha)) = g(kt)e^{-\alpha kt}$$

It then follows that the process impulse response $g(kT)$ is

$$g(kT) = g(kt)e^{-\alpha kt}$$

Applying the FFT again to $g(kT)$ leads to the process frequency response:

$$G(jw) = \text{FFT}(g(kT)) \quad (6)$$

The method can accurately identify as many as desired frequency response points with one relay experiment. They may be very useful for improving the performance of PID and other model-based controllers. The required computations are more involved than the standard relay technique, especially if a large number of frequency response points are needed. In both applications: PID tuning and transfer function modeling, the shifted frequency response may be used without the needing to computer $G(jw)$. To illustrate the method, a model with oscillatory dynamics is considered in simulation.

$$G(s) = \frac{1}{s^2 + 2s + 1} e^{-2s} \quad (7)$$

Figure 1 shows the identified frequency responses for these processes using this method, for $G(jw)$. And $G(jw+\alpha)$ plot is given by figure 2.

### 2.3 Simple Genetic Algorithms

The genetic algorithm is a useful tool to solve both constrained and unconstrained optimization problems that takes principles of biological evolution [16-19]. The following procedure summarizes the main steps that the genetic algorithm executes:
1. The algorithm starts by creating a random initial population.
2. The algorithm then creates a sequence of new populations. At each step, the algorithm uses the individuals in the current generation to create the next population. To create the new population, the algorithm performs the following steps:
   a. Assign a grade to each member of the current population by computing its fitness value.
   b. Selects members, called parents, based on their fitness.
   c. Some of the individuals in the current population that have better grades are chosen as elite. These elite individuals are passed on to the next population.
   d. Children from the parents are produced by means of crossover and mutation operators.
   e. The current population is replaced with the children to form the next generation.
3. The algorithm stops when one of the stopping criteria is met.

3 APPLICATIONS

3.1 PID Tuning via frequency response fitting

PID Tuning via frequency response fitting is a simple but efficient solution to this kind of processes that was developed [4]. It shapes the loop frequency response to optimally match the desired dynamics over large range of frequencies. Thus the closed-loop performance is more firmly guaranteed than in the case of only one or two points PID or PI tuning laws. Suppose that multiple process frequency response points \( G(j\omega_i), i=1,2,...,m \), are available. The control specifications can be formulated as a desirable closed loop transfer function

\[
H_d(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} e^{-Ls}
\]

where \( L \) is the apparent dead-time of the process, \( \omega_n \) and \( \zeta \) dominate the behavior of the desired closed-loop response, [4]. If the control specifications are given as the phase margin \( \Phi_m \), and gain margin \( A_m \) \( \omega_n \) and \( \zeta \) in \( H_d \) are approximately determined by

\[
\zeta = \sqrt{1 - \cos^2(\Phi_m)}
\]

and

\[
\omega_n = \frac{\tan^{-1}(\frac{2\zeta p}{p^2 - 1})}{pL}
\]

where \( p \) is the positive root of equation

\[
(A_m-1)^2 = 4\zeta^2 p^2 + (1-p^2)^2
\]

The default settings for \( \zeta \) and \( \omega_n \) values are \( \zeta = 0.707 \) and \( \omega_n L = 2 \), which imply that the overshoot of the objective set-point step response is about 5%, the phase margin is 60° and the gain margin is 2.2 [20]. The open-loop transfer function corresponding to \( G_d \) is

\[
G_d = \frac{H_d}{1-H_d}
\]

The controller \( C(j\omega) \) is designed such that the actual \( GC(j\omega) \) is fitted to the desired transfer function \( G_d(j\omega) \), as well as possible. Thus the resultant system will have the desired performance. The PID controller desired can be obtained by minimizing the objective function given from the sum of squared differences between computed and recorded frequency response points

\[
CG(j\omega_i) = \frac{K_p(j\omega_i + Ki + Kd(j\omega_i))}{j\omega_i} G(j\omega_i)
\]

\[
G'_d(j\omega_i) = \left[ \begin{array}{c}
\text{Real}(G_d(j\omega_i)) \\
\text{Imag}(G_d(j\omega_i))
\end{array} \right]
\]

\[
G''_d(j\omega_i) = \left[ \begin{array}{c}
\text{Real}(GC(j\omega_i)) \\
\text{Imag}(GC(j\omega_i))
\end{array} \right]
\]

The objective function

\[
y = \sum_{i=1}^{m} |G''_d(j\omega_i) - G'_d(j\omega_i)|^2
\]

If the PID controller is designed from \( G(j\omega + \alpha) \), then

\[
C(j\omega_i + \alpha) = \frac{K_p(j\omega_i + \alpha) + Ki + Kd(j\omega_i + \alpha)}{}^2
\]

\[
CG(j\omega_i + \alpha) = C(j\omega_i + \alpha) G(j\omega_i + \alpha)
\]

\[
G'_d(j\omega_i + \alpha) = \left[ \begin{array}{c}
\text{Real}(G_d(j\omega_i + \alpha)) \\
\text{Imag}(G_d(j\omega_i + \alpha))
\end{array} \right]
\]

\[
G''_d(j\omega_i + \alpha) = \left[ \begin{array}{c}
\text{Real}(GC(j\omega_i + \alpha)) \\
\text{Imag}(GC(j\omega_i + \alpha))
\end{array} \right]
\]
The objective function

\[ y = \sum_{i=1}^{m} |CG'(j\omega_i + \alpha) - G'_d(j\omega_i + \alpha)|^2 \]  

(13)

The solution of the problem is obtained by minimizing \( y \).

In this work, the identified points were obtained from a schematic Simulink® system where relay feedback system is simulated. To solve the optimization problem, the MATLAB® genetic algorithm optimizations using the optimization tool GUI is used.

**Example 1.** Consider a model with oscillatory dynamics

\[ G(s) = \frac{1}{s^2 + 2.5s + 1} e^{-2s} \]  

(14)

The identified points for this model are showed in figures 1 and 2. In this example the apparent dead-time \( L=0.2 \), is proposed.

The designed PID is solved by minimizing the equation 11 by means of a simple genetic algorithm. Multiple points are from \( G(j\omega) \)

\[ C(s) = (0.598 + \frac{2.94}{s} + 2.96s) \]  

(15)

And from \( G(j\omega + \alpha) \), the tuned PID is

\[ C(s) = (0.6 + \frac{2.94}{s} + 2.998s) \]  

(16)

Eq. (15)-(16) show that both PID’s controllers have very close values as might be expected.

Performance of the PID designed is shown in the figure 4.

**Example 2.** Considerer a high order model

\[ G(s) = \frac{1}{(s+1)^{10}} \]  

(17)

For this model the value of **apparent dead-time** of the process \( L=4.5 \) was proposed.

Estimated model from \( G(j\omega) \) the design PID is

\[ C(s) = (0.922 + \frac{0.149}{s} + 2.62s) \]  

(18)

Performance of the PID designed is shown in figure 5.

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**3.2 The sensitivity to modeling errors**

Since the controller is tuned for a particular process, it is desirable that the closed loop system is not very sensitive to variations of the process dynamics [21]. A convenient way to express the sensitivity of the closed loop system is through the sensitivity function \( S(s) \), defined as: \( S(s) = \frac{1}{1 + L(s)} \), where \( L(s) \) denotes the loop transfer function [2, 3,5,7,22]. \( L(s) \) is given by:

\[ L(s) = C(s)G(s) \]

\[ = G(s)\left(1 + \frac{1}{T_S} + T_C s\right) \]

The maximum sensitivity (frequency response) is then given by \( M_s = \max|S(j\omega)| \). Therefore \( M_s \) is given by \( \|S(s)\|_{\infty} \). On the other hand, it is known that the quantity \( M_p \) is the inverse of the shortest distance from the Nyquist curve of loop transfer function to the critical point \( s=-1 \) [2]. Typical values of \( M_s \) are in the range from 1.2 to 2.0.

Table 1 shows the values of \( M_s, M_p \) and \( \Phi_m \) for both presented examples, model with oscillatory dynamics and high order model.

The operation of genetic algorithm was configured with the following parameter values:
• Population size: 100
• Stochastic uniform Selection
• Crossover function: Scattered
• Mutation function: Gaussian
• Number of generation: 500
• Crossover probability: 0.8
• Mutation probability: 0.09
• Elite count: 2

3.3 Transfer function modeling

A transfer function model is necessary in many applications of automatic control. In this work a second order plus dead-time model is proposed. The identification at models with dead-time is usually a non-linear problem [4, 8, 23]. This characteristic presents a good opportunity to apply a genetic algorithm to solve the problem.

\[
G(s) = \frac{1}{as^2 + bs + c} \tag{19}
\]

Which can represent both monotonic and oscillatory processes.

3.3.1 Transfer function modeling from \( G(jw) \)

Suppose the process frequency response \( G(jw_i), i=1,2,...,M \) is available, because they are required to be fitted into \( G(s) \) in (19) such that

\[
G_m(jw_i) = \frac{1}{a(jw_i)^2 + bjw_i + c} e^{-Lw_i} \tag{20}
\]

where \( i=1,2,...,M \)

then

\[
G_m'(jw_i) = \left[ \begin{array}{c}
\text{Real}(G_m(jw_i)) \\
\text{Imag}(G_m(jw_i))
\end{array} \right] \]

And the identified points of \( G(jw) \)

\[
G'(jw_i) = \left[ \begin{array}{c}
\text{Real}(G(jw_i)) \\
\text{Imag}(G(jw_i))
\end{array} \right] \]

The objective function is

\[
y = \sum_{i=1}^{m} |G_m(jw_i) - G'(jw_i)|^2 \tag{21}
\]

The solution of the problem is obtained by

\[
\min \sum_{i=1}^{m} |G_m(jw_i) - G'(jw_i)|^2 \tag{22}
\]

3.3.2 Transfer function modeling from \( G(jw+\alpha) \)

Suppose the shifted frequency response of the process \( G(jw_i + \alpha), i=1,2,...,M \) is available, because they are required to be fitted into \( G(s) \) in (19) such that

\[
G_m(jw_i + \alpha) = \frac{1}{a(jw_i + \alpha)^2 + bjw_i + \alpha + c} e^{-L(w_i + \alpha)} \tag{23}
\]

where \( i=1,2,...,M \)

then

\[
G_m'(jw_i + \alpha) = \left[ \begin{array}{c}
\text{Real}(G_m(jw_i + \alpha)) \\
\text{Imag}(G_m(jw_i + \alpha))
\end{array} \right] \]

And the identified points of \( G(jw) \)

\[
G'(jw_i + \alpha) = \left[ \begin{array}{c}
\text{Real}(G(jw_i + \alpha)) \\
\text{Imag}(G(jw_i + \alpha))
\end{array} \right] \]

The objective function is

\[
y = \sum_{i=1}^{m} |G_m(jw_i + \alpha) - G'(jw_i + \alpha)|^2 \tag{24}
\]

The solution of the problem is obtained by

\[
\min \sum_{i=1}^{m} |G_m(jw_i + \alpha) - G'(jw_i + \alpha)|^2 \tag{25}
\]

Examples

Table 2 shows the results of some examples that were proposed to obtain the identified models from multiple points from \( G(jw) \) and \( G(jw+\alpha) \). The estimated models were solved by minimizing the equations (22) and (25) by means of a simple genetic algorithm.
3.4 Transfer function modeling for processes with long dead-time

Processes with long dead-time are present in most of the industrial processes and can be adequately approximated by a model in form of

\[ G(s) = \frac{K}{(Ts+1)^2} e^{-Ts} \]  

(26)

Example 3 consider a high vacuum distillation column which is a typical long dead-time process [9]

\[ G(s) = \frac{0.57}{(8.6s+1)^2} e^{-18.7s} \]  

(27)

The identified model is given in Eq. (28). It was obtained by the same method as was presented previously. Figure 6 shows the identified points on the Nyquist curve from \( G(jw+\alpha) \).

\[ \hat{G}(s) = \frac{0.571}{(8.6s+1)^2} e^{-18.71s} \]  

(28)

In this example, the number of generations and population size used for genetic algorithm are: 1500 and 100 respectively.
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